# GROUND WATER

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# Estimation of Ground-Water Mounding Beneath Septic Drain Fields

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## Abstract

Localized ground-water mounding beneath larger on-site sewage disposal fields (septic drain fields) can reduce and even eliminate the waste-water treatment that occurs in the unsaturated soil zone. Such mounding was previously predicted for longer times by a procedure of uncertain accuracy and having a number of limitations. In this paper, a new procedure is developed on a stronger theoretical basis. The governing equation may be solved by four different methods. The new procedure considerably extends the time range of applicability, and no longer restricts the proportions of the rectangular disposal field. Errors in the previous methods approached 20% in some cases. Comparisons of different methods to reduce mound heights where they are critical indicate that the most effective method is to subdivide the disposal field into separated subareas.

## Introduction

About one-third of U.S. homes are served by septic drain fields, also known as filter fields, leach fields, soil absorption systems, and on-site sewage disposal systems. They largely occur in urban fringe and rural residential areas, and also commonly serve rural institutional buildings and recreational developments. Dependence on such systems has grown as a result of their being increasingly viewed as permanent rather than interim facilities. Both this increasing dependence and the general growth of environmental awareness have resulted in greater concern that such disposal systems may be contributing to long-term ground-water pollution.

Authorities responsible for environmental health strive to protect ground-water quality by specifying a minimum setback, or vertical distance, between the bottom of the disposal trenches or beds and the water table; this is typically from 2-5 ft deep. In this unsaturated soil zone occur relatively high levels of physical, biological, and chemical treatment.

Depending on the type of soil, and on the design and use of the disposal system, the water table beneath the discharge area could rise enough to reduce the unsaturated zone depth and the treatment it provides, or even short-circuit it entirely. Therefore, it is vitally important that designers and regulators of such disposal systems should have reliable methods to estimate the water-table rise or mounding that might occur over the life of the facility, and that they should be able to use these methods to identify designs which reduce mounding where necessary.

The purposes of this paper are to provide an improved procedure for the prediction of long-term ground-water mounding beneath septic drain fields, and to suggest further design strategies to reduce mounding.

## Ground-Water Mounding

Here we shall consider the common situation where a disposal field drains down into, and forms a ground-water mound on, an extensive and initially near-horizontal saturated zone (Figure 1). Various methods for analyzing this situation have been reviewed and/or summarized in a number of publications (Finnemore and Hantzsche, 1983; Hensel et al., 1984; Urish, 1991). The majority of these procedures either follow or are based on the method of Hantush (1967), mainly because it is more generalized and more accurate than the other methods.

The analysis is greatly simplified by limiting it to the highest, and therefore most critical, point on the ground-water mound.

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## Previous Simplified Long-Term Mounding Prediction

From the Hantush (1967) procedure for predicting ground-water mound heights, the rise of the highest point at the center of the mound can be expressed as

$$z_m = \frac{ItS^*}{S_y} \tag{1}$$

in which I = average volume recharge rate of waste-water entry into unit area (e.g., 1 $ft^2$ or 1 $m^2$) of soil; t = time since the beginning of waste-water application; $S_y$ = specific yield of aquifer, which is the volume fraction of the total aquifer which will drain freely; and S* is a tabulated function of $\alpha$ and $\beta$. Here

$$\alpha = \frac{L}{4}\sqrt{S_y/(K\bar{h}t)} \tag{2a}$$

$$\beta = \frac{W}{L}\alpha \tag{2b}$$

in which L and W = respectively, the length and width of the disposal field (waste-water application area; $L \geq W$); K = horizontal hydraulic conductivity of the aquifer; and

$$\bar{h} = h_0 + \tfrac{1}{2} z_m \tag{3}$$

Besides the inconvenience of the need to look up tabulated values, usually requiring interpolation, the tabulated values limit applications to maximum rise times of about 2-8 years, depending on parameter values.

For longer times, t, on the order of 10-40 years, which are of particular concern with disposal fields, the magnitudes of $\alpha$ and $\beta$ become very small. In order to approach the origin more closely where tabulated values of $\alpha$ and $\beta$ are unavailable, Finnemore and Hantzsche (1983) fitted the tabular function for S* by expressions of the form $S^* = C\alpha^n$ in the region where $\alpha$ and $\beta$ are $\leq 0.04$. They presented values of the constants C and n for length-to-width ratios (L/W) of 1, 2, 4, and 8. This form of expression for S* enabled equations (1) and (2a) to be combined, leading to their equation for longtime mound height

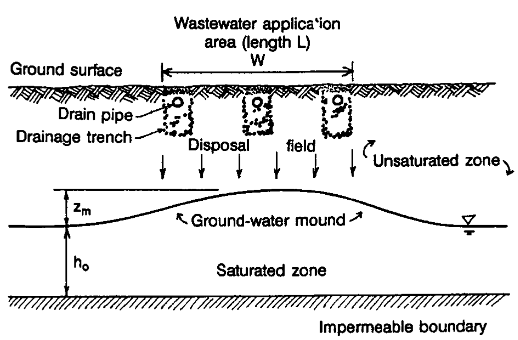


**Fig. 1. Ground-water mound beneath rectangular disposal field (modified from Finnemore and Hantzsche, 1983).**

$$z_m = IC\left(\frac{L}{4}\right)^n \left(\frac{1}{K\bar{h}}\right)^{0.5n} \left(\frac{t}{S_y}\right)^{1-0.5n} \tag{4}$$

The fact that these fits were made for $\alpha \leq 0.04$ led to the requirement that $t > t_{min} = 40L^2 S_y/(Kh_0)$ for results to be accurate.

Equation (4) provided for the first time an ability to predict maximum mound heights at long times. As a result, equation (4) has been used by state regulatory agencies concerned with septic drain field design (Hensel et al., 1984; Urish, 1991). It is not straightforward to solve (4) for $z_m$ given t, however, because $z_m$ is included in $\bar{h}$. Various solution methods have subsequently been suggested by Finnemore (1992).

## New Simplified Long-Term Mounding Prediction

Although the smallest tabulated values of S* given by Hantush (1967) were for $\alpha$ and $\beta$ equal to 0.02, he stated in his equation (25) that

$$S^* \approx \frac{4}{\pi}\alpha\beta\left\{3 - \left[\frac{\alpha}{\beta}\tan^{-1}\frac{\beta}{\alpha} + \frac{\beta}{\alpha}\tan^{-1}\frac{\alpha}{\beta}\right] + E(u)\right\} \tag{5}$$

provided $u = (\alpha^2 + \beta^2) \leq 0.10$; this approximation becomes more accurate as $u \to 0$. Here E(u) is the exponential integral of u; for specific values of u it is known to hydrologists as the well function of u, W(u).

From (2b) we note that $\alpha/\beta = L/W$; letting the length-to-width ratio $L/W = r$, we can write

$$\frac{\alpha}{\beta}\tan^{-1}\frac{\beta}{\alpha} + \frac{\beta}{\alpha}\tan^{-1}\frac{\alpha}{\beta} = r\tan^{-1}\frac{1}{r} + \frac{1}{r}\tan^{-1} r = J(r) \tag{6}$$

so that (5) becomes

$$S^* \approx \frac{4}{\pi}\alpha\beta\{3 - J(r) + E(u)\} \tag{7}$$

where we find that $(\pi/2) \geq J(r) > 1$ because $1 \leq r < \infty$.

Substituting for S* from (7) into (1), while also substituting for $\alpha$ and $\beta$ from (2), we obtain

$$z_m \approx \frac{L^2 I}{4\pi r K\bar{h}}\{3 - J(r) + E(u)\} \tag{8}$$

If we let Q be the average volume recharge rate of waste-water entry into the entire disposal field, then $Q = LWI = L^2 I/r$ and (8) can be written

$$z_m \approx \frac{Q}{4\pi K\bar{h}}\{3 - J(r) + E(u)\} \tag{9}$$

E(u) may be written as the series expansion

$$E(u) = -\gamma - \ln u + u - \frac{u^2}{4} + \ldots + \frac{(-1)^n u^n}{n.n!} \tag{10}$$

Here $\gamma$ is Euler's constant = 0.5772156649. We notice in (10) that as $u \to 0$, the polynomial terms $\to 0$ rapidly, and so

$E(u) \rightarrow -\gamma - \ln u$. Earlier we noted that this equation requires $u \leq 0.10$. So writing the sum of the small polynomial terms after ln u in (10) as $\epsilon(u)$, substituting for $E(u)$ from (10) into (9), and rearranging a little yields

$$\frac{4\pi K}{Q} z_m \bar{h} \approx 3 - \gamma - J(r) - \ln u + \epsilon(u) \qquad (11)$$

Evaluation of the right side of equation (11) for various values of u reveals that $\epsilon(u)$/(right side) is less than 3.1% for $u \leq 0.10$, and less than 0.7% for $u \leq 0.03$. Therefore, for the application considered here, the terms represented by $\epsilon(u)$ may be neglected when $u \leq 0.03$, when the approximation involved is also more accurate, so that a new governing equation may be written as

$$z_m = \frac{Q}{4\pi K\bar{h}} \{3 - \gamma - J(r) - \ln u\} \qquad (12)$$

Making use of equations (2) and our definition of r, (12) is valid provided

$$u = (\alpha^2 + \beta^2) = \alpha^2\left(1 + \frac{1}{r^2}\right) = \frac{L^2 S_y}{16K\bar{h}t}\left(1 + \frac{1}{r^2}\right) \leq 0.03$$

which, by noting that $[1 + (1/r^2)] \leq 2$ and $h_0 \leq \bar{h}$, leads to the requirement that $t > t_{min} = 4.17L^2 S_y/(Kh_0)$ for results from equation (12) to be accurate.

If $z_m$ is given, and one wishes to solve for t, which is included in u, equation (12) can be rearranged into

$$t = \frac{L^2 S_y}{16K\bar{h}}\left(1 + \frac{1}{r^2}\right) \exp\left[\frac{4\pi K}{Q} z_m \bar{h} - 3 + \gamma + J(r)\right] \qquad \ldots (13)$$

However, we usually wish to solve for $z_m$ at prescribed times. In this case, solving for $z_m$ is not straightforward because it is included in both $\bar{h}$ and u in such a way that it cannot be separated out. Four different methods of solving equation (12) for $z_m$ given t are now suggested.

**Method 1.** An iterative procedure may be used, starting with a value of $z_m$ estimated from experience or from an approximate application of method 3 below; better first estimates of $z_m$ will reduce the number of iterations needed. The estimated value of $z_m$ is used to calculate $\bar{h}$ and u, and the right side of equation (12) then yields an improved estimate of $z_m$. If this is not the same as the assumed value, the process must be repeated.

**Method 2.** Iterations can be greatly reduced by the use of Newton's method. With this applied to equation (12), a better estimate of $z_m$ is

$$z_{m1} = z_m - \frac{z_m(h_0 + \frac{1}{2}z_m) - 2B[3 - \gamma - J(r) - \ln u]}{(h_0 + z_m) - B/(h_0 + \frac{1}{2}z_m)} \qquad \ldots (14)$$

where $B = Q/(8\pi K)$. This converges far more rapidly.

**Method 3.** Iteration can be avoided all together as follows. Setting $z_m/h_0 = \xi$, equation (12) can be rearranged into the form

$$\frac{\xi(1 + \frac{1}{2}\xi)}{3 - \gamma - J(r) - \ln\left[N[1 + (1/r^2)]/(1 + \frac{1}{2}\xi)\right]} = \frac{Q}{4\pi K h_0^2} \qquad (15)$$

where $N = L^2 S_y/(16Kh_0 t)$ which is dimensionless; for t to be greater that $t_{min}$, N must be $< 1.498 \times 10^{-2}$. If we call the right side of (15) R, which is also dimensionless, then both N and R contain all known quantities. So a graph of $\xi$ vs R, plotted for various values of r and N, as in Figur. 2, enables $\xi$ (and so $z_m$) to be found directly when R, r, and N are known. The accuracy of this method is limited by the accuracy of reading and interpolating from the graph, but it is fast. It also provides a good initial estimate to speed up methods 1 and 2.

**Method 4.** Equation (12) can be solved accurately and without manual iteration by use of a programmable scientific calculator which has an equation (root) solving capability, as on the Hewlett-Packard 42S and 48S calculators. With such calculators the equation must first be entered and stored permanently; if necessary it can be corrected by editing. Values of each of the known variables in the equation are then stored in memory locations. Then the root solving capability is activated to solve for the unknown variable; internal programs cause the calculator to perform the iterations needed to find the value of the unknown variable, in our case $z_m$, which satisfies the equation. Once installed and verified, this is the most convenient and accurate method.

Values of $z_m$ obtained by methods 2-4 can always be checked by using them to evaluate the right side of equation (12), which should of course equal the left side.

Examples of the growth of mound height with time are presented as solid curves in Figures 3-4 for various soil horizontal conductivities. Below the limit for accuracy, i.e.

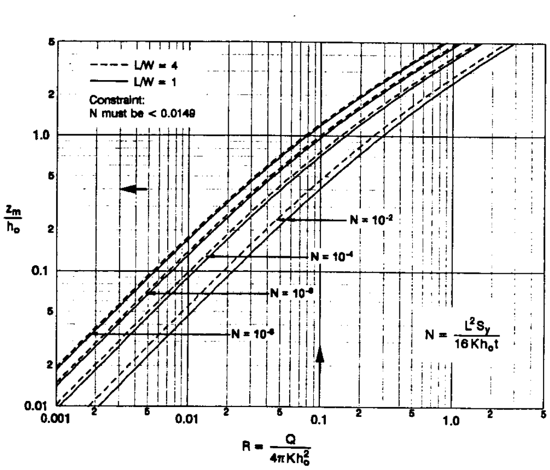


**Fig. 2. Graphical solution of equation (12).**

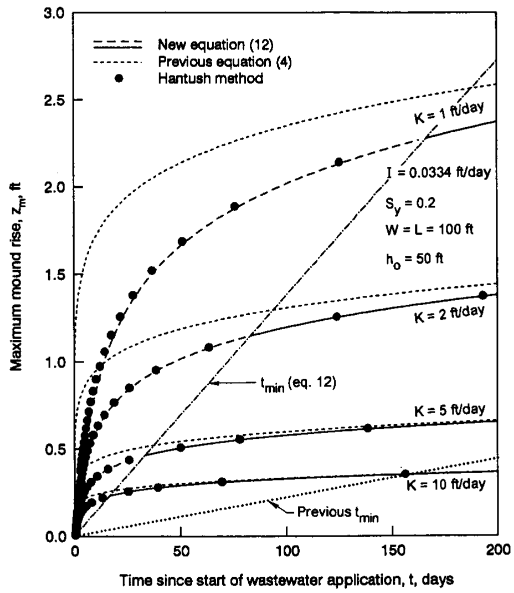


**Fig. 3. Example mound growths at short waste-water application times (adapted from Finnemore and Hantsche, 1983).**

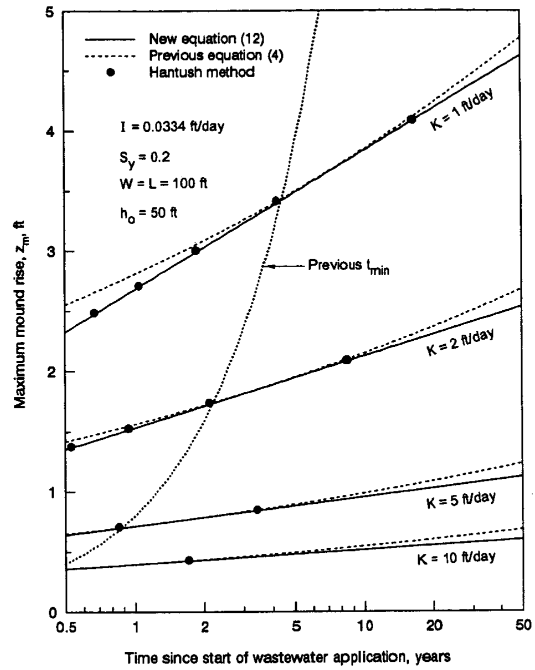


**Fig. 4. Example mound growths at long waste-water application times (adapted from Finnemore and Hantzsche, 1983).**

for $t < t_{min}$, the curves are shown dashed. They are compared with accurate results computed from Hantush's tabulations presented as solid circles, and with previous equation (4) shown as dotted curves.

We observe in Figures 3-4 that a mound continues to grow without limit, though at ever reducing rates. The curves of Figure 4 may be misleading in this regard, as they are distorted by the logarithmic time scale used; and although they may appear to be straight, they are slightly curved downwards. The unending mound growth follows from the Hantush equations and tabulations, and results from his assumption of an infinite aquifer. Because actual aquifers have outlets at finite distances, the mound growth will be limited and estimates given by the Hantush procedure and the above equations will be safe estimates on the high side.

It is important to remember that the mound height calculation method described here, and the Hantush equation upon which it is based, calculates the effect of only a singlc septic drain field, and so does not consider the cumulative effects of multiple, interfering drain fields. At higher development densities these cumulative effects will certainly increase mound heights.

## Comparisons of Predictions and Methods

Before the above new simplified method was developed, the accuracy achieved by using the fitted equation $S^* = C\alpha^n$ was not known for longer times. Some differences can be seen on Figures 3 and 4. Considering new equation (12) to be correct for $t > t_{min}$, more examples of differences predicted by previous equation (4) are presented in Figure 5. Percentage errors are seen to vary with time, and to approach 20% in some instances. In most cases the errors were on the high side, overpredicting mound heights. Although such high predictions by equation (4) may be viewed as being on the safe side, in certain tight design situations they might unnecessarily disqualify a valid solution. Fortunately, most of the larger percentage errors were found to occur when mound heights are smaller.

There are four notable advantages of the new prediction method described here, in comparison to previous

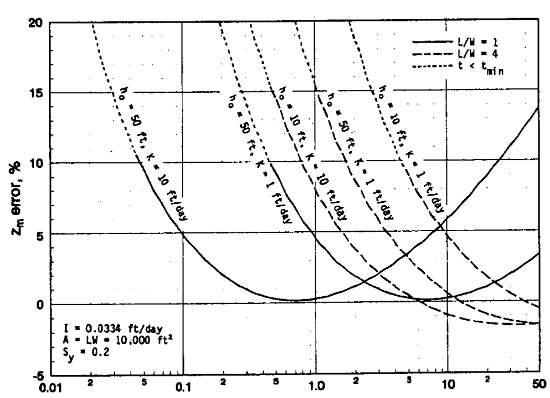


**Fig. 5. Example differences between equations (4) and (12).**

equation (4). These advantages are as follows. First, the new method does not use any fitted functions, and so avoids any fitting errors such as those just described. Second, the nature of all its approximations to the Hantush procedure are quite apparent, so that their magnitudes can be readily estimated. Third, the new method can use any L/W ratio; the previous method was limited to ratios of 1, 2, 4, and 8. Fourth, the minimum time limit for reliability with the new method is about one-tenth of that for the previous method (comparing coefficients of 4.17 and 40). This greatly extends the range of accuracy to shorter times. This is particularly helpful for drain fields with larger L/W ratios since they have large accuracy limits, $t_{min}$, because these are proportional to $L^2$. For larger drain fields with larger L/W ratios, low soil hydraulic conductivities, and shallow saturated zones, $t_{min}$ for the previous method could easily exceed 50-100 years, which made it of little practical utility in such cases. With the new method, these time limits become 5-10 years, making it very usable.

In addition to these advantages, equation (12) is usually more convenient to use than (4). Instead of evaluating three quantities raised to different exponents, there are two arctangents and one logarithm to evaluate; the two arctangents (in J) change only with changing L/W ratio.

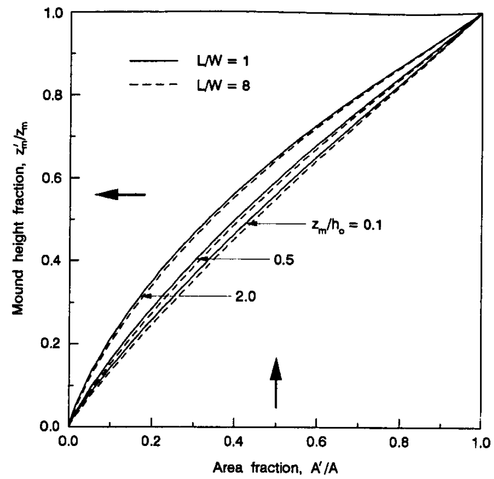


**Fig. 6. Influence on mound growth of dividing up and widely separating disposal fields.**

## Design Considerations

When undertaking disposal field design, a good appreciation of the respective impacts of the various parameters involved is required.

Examples of the effects of hydraulic conductivity K and time t on mound height $z_m$ are shown in Figures 3 and 4. There we may see that mounds grow much larger for smaller K values, which impede the spreading of the extra water. It is important to remember that the K values cited are in the horizontal direction, which for many soils may be significantly larger than the vertical conductivities. The strong influence of the saturated zone depth $h_0$, particularly when shallow, and the relatively minor effect of specific yield $S_y$, have been described by Finnemore and Hantzsche (1983).

Methods of determining the aquifer properties ($h_0$, K, $S_y$) required for prediction and design are discussed by Finnemore and Hantzsche (1983). They noted that the accuracy of the prediction methods depends in particular on the accuracy of the determination of K and $h_0$.

When mound heights are critical, four different design options for reducing them are: drain field elongation (of a given area); drain field enlargement, which reduces I; intermittent drain field operation; and drain field subdivision. The effects of the first three of these options are discussed more fully by Finnemore and Hantzsche (1983); in summary, mound height reductions obtained by the first two are minor, and by the third are negative. The effect of subdividing a single disposal field into a number of widely separated smaller fields, each with the same I and L/W proportions as the original, may be found more conveniently from equation (4) than from (12). Using a prime to indicate parameters for one of the smaller fields, we may write (4) twice, for the original field and for the smaller field with primes, and take the ratio of the two equations to obtain the ratio of the two mound heights. Many quantities cancel because they are the same in both equations. After some rearrangement, and noting that field area A is proportioned to $L^2$, we obtain

$$\frac{A'}{A} = \left(\frac{z_m'}{z_m}\right)^{2/n}\left(\frac{h_0}{z_m} + \frac{1}{2}\frac{z_m'}{z_m}\right) \Big/ \left(\frac{h_0}{z_m} + \frac{1}{2}\right) \qquad (16)$$

From this, the mound height fraction, $z_m'/z_m$, may be plotted against the area fraction, A'/A, for various $z_m/h_0$ and L/W ratios. These plots are presented in Figure 6, which reveals that the $z_m/h_0$ ratio has a minor effect on results, and that the L/W ratio has a very slight effect. More importantly, it demonstrates that subdividing a single disposal field into widely separated, smaller fields reduces mound heights far more effectively than the other three options. For example, Figure 6 indicates that replacing a single field by two widely separated fields each with half the area reduces the mound height to 55-65%.

As noted earlier, the amount of mound height reduction achieved by disposal field subdivision depends upon the amount of separation; the amounts indicated by Figure 6 are the greatest reductions obtainable, when the mounds are sufficiently widely spaced to have negligible effects upon one another. The interactive effects of less widely spaced mounds are not presently known. However, as two widely spaced mounds are brought closer together, and the interaction effects become stronger and lessen the height reduction due to subdivision, the limiting case will be when they make contact and become a single, elongated mound. The height reduction for this limiting case can be calculated from equation (12), and it is known to be minor (Finnemore and Hantzsche, 1983). In this way mound height reductions for intermediate spacings may be bracketed, and it is clear that they must be greater than the minor reductions obtainable by drain field elongation or enlargement.

Practical applications to design are discussed at some length by Finnemore and Hantzsche (1983), and so need little repetition here. Notably, 20-year mound heights for individual homes are unlikely to exceed one foot except where aquifers are very shallow ($h_0 < 10$ ft) and soil conductivities are very low (i.e., $K \approx 1$ ft/day, which is marginally acceptable). Larger mounds of concern are more likely to occur beneath disposal fields serving clusters of homes, institutional buildings, or recreational developments. Flow rates Q and I used in calculations should be average values because mounding is a long-time cumulative effect. A 20-year life is recommended for design, as suggested by Urish (1991), because it is representative of the life of the facility; if it should serve for 40 years, the mound could grow by only another 7-8%.

## Summary and Conclusions

Ground-water mounding beneath on-site sewage disposal fields, particularly larger ones, can threaten the wastewater treatment that occurs in the unsaturated soil zone. This paper presents an improved procedure for predicting longer-time mound heights, having a stronger theoretical basis.

The new procedure considerably extends the range of applicability to shorter times. In some cases the time limit of applicability of the only available former method was so large as to invalidate it for the normal service lifetimes of such facilities. Also, the new procedure no longer restricts the rectangular disposal field to a few specified proportions; any length-to-width ratio may now be used. The accuracy of the former prediction method was previously unknown. Comparisons with the new procedure indicate that the former method usually overpredicts mound heights, by amounts that may approach 20% in some instances.

Mound heights will be greatest in shallow aquifers of low permeability. The most effective method to reduce mound heights is to divide the disposal field into subareas, the more widely separated the better. Field enlargement or elongation achieves relatively slight reductions, and intermittent field operation increases maximum mound heights.

As noted by Finnemore and Hantzsche in 1983, such prediction methods need to be used with judgment by experienced engineers who are aware of the limitations, and there continues to be a need for field verification.

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